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Quadratic action for type IIB supergravity on $AdS_5 \times S^5$

Gleb E. Arutyunov[a,c] **and Sergei A. Frolov**[b,c]

[a] *Dipartimento di Matematica, Universita di Milano, "Federigo Enriques" Via C.Saldini, 50-20133 Milano, Italy*

[b] *Department of Physics and Astronomy, University of Alabama, Box 870324, Tuscaloosa, Alabama 35487-0324, USA*

[c] *Steklov Mathematical Institute, Gubkin str.8, GSP-1, 117966, Moscow, Russia*

E-mail: arut@genesis.mi.ras.ru, frolov@bama.ua.edu

Abstract: The quadratic action for physical fields of type IIB supergravity model on $AdS_5 \times S^5$ is derived starting from the recently found covariant action. All boundary terms that have to be added to the action to be used in the AdS/CFT correspondence are determined.

Keywords: String Duality, Superstring Vacua, Classical Theories of Gravity.

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Contents

1. Introduction

It is well-known that the covariant equations of motion for type IIB supergravity [1, 2, 3] can not be derived from any action because of the presence of a self-dual five-form. However, after eliminating some unphysical fields, one arrives at equations of motion which are not manifestly covariant but admit a lagrangean description. The formal covariance of the lagrangean can then be provided by introducing auxiliary non-propagating fields. This idea was successfully applied in [4, 5] to construct a covariant action for type IIB supergravity. The covariance of the action has to be taken with a grain of salt since one cannot impose any covariant gauge conditions on the auxiliary fields. Nevertheless, the very existence of the action allows one to study in detail the properties of supergravity. The existence of a covariant action for type IIB supergravity has special interest due to the discovery of the duality between type IIB superstring theory on the $AdS_5 \times S^5$ background and the four-dimensional $\mathcal{N} = 4$ SU(N) super Yang-Mills model [6]. As was argued by Maldacena, in the large N limit and in the limit of large t'Hooft coupling $\eta = g_{YM}^2 N$ the SYM model may be described by the classical type IIB supergravity on $AdS_5 \times S^5$. In particular, the physical fields of supergravity correspond to local primary operators of the SYM model.

The conjecture by Maldacena was further elaborated by Gubser, Klebanov and Polyakov [7] and by Witten [8], who proposed that the generating functional of the connected Green functions in the SYM model coincides with the minimum of the supergravity action subject to certain conditions imposed on supergravity fields[1] at the boundary of $AdS_5 \times S^5$. It is worth noting that to make the AdS/CFT correspondence complete one has to add to a supergravity action boundary terms. The origin of the boundary terms was recently clarified in [9], where it was shown that they appear in passing from the hamiltonian formulation of the supergravity to the lagrangean one.

The AdS/CFT correspondence has already been used in [10]–[24] to compute some two- and three-point Green functions up to normalization constants, and some preliminary results on four-point Green functions have also been obtained [25, 26, 27]. However, a detailed investigation of the AdS/CFT correspondence requires the knowledge of the type IIB supergravity action. In particular, to fix the normalization constants of two- and three-point Green functions one has to know the quadratic and cubic actions for physical fields of supergravity. To this end one may try to use the covariant action by [4, 5]. In a recent paper [18] the quadratic action for scalar fields corresponding to the chiral primary operators in the $\mathcal{N} = 4$ SYM was found by comparing the on-shell values of the covariant action and the most general quadratic action for the scalar fields. This method cannot be used to determine the complete quadratic type IIB supergravity action, since for some fields, e.g. fermions, the on-shell value of the action vanishes, if one does not take into account boundary terms which are in general unknown. Thus the only way to find the quadratic action is to derive it directly from the covariant one.

The aim of the present paper is to determine the bulk quadratic action for physical fields of type IIB supergravity. Then, by using the approach of [9] we find all boundary terms, that have to be added to the bulk action in order to get the action that has to be used in the AdS/CFT correspondence.

The plan of the paper is as follows. In section 2 we briefly discuss the covariant action of [4, 5] and calculate the quadratic covariant action including all (physical and unphysical) fields of type IIB supergravity. In section 3 we consider the part of the action that depends on the gravitational fields and the 4-form potential and, in particular, reproduce the result of [18]. However, contrary to the results of [18], there is no nonlocality in the action. The antisymmetric tensor fields are considered in section 4, where we represent the corresponding action in the first-order formalism. The quadratic action for fermions is obtained in section 5. In the Conclusion we discuss some possible applications of the results obtained and unsolved problems. The mass spectrum we obtain coincides with the one found in [28, 29].

[1] The boundary conditions of [7, 8] can be imposed only on the supergravity physical fields that satisfy second-order differential equations.

2. Covariant action

The covariant action of [4, 5] for type IIB supergravity can be written in the form:

$$
\begin{aligned}
S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\Big(& R - \frac{4}{5!}F_{M_1\ldots M_5}F^{M_1\ldots M_5} + \frac{1}{3!}\frac{\partial^L a\partial_K a}{(\partial a)^2}\mathcal{F}_{LM_1\ldots M_4}\mathcal{F}^{KM_1\ldots M_4} + \\
& + \frac{2}{5!}\varepsilon^{M_1\ldots M_{10}}F_{M_1\ldots M_5}B_{M_6M_7}\partial_{M_8}C_{M_9M_{10}} - \\
& - 3\mathrm{e}^{-\varphi}(\partial_{[M}B_{NK]})^2 - 3\mathrm{e}^{\varphi}(\partial_{[M}C_{NK]} - \chi\partial_{[M}B_{NK]})^2 - \\
& - \frac{1}{2}(\partial_M\varphi)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}(\partial_M\chi)^2\Big) + S(\psi)\,,
\end{aligned}
\tag{2.1}
$$

where $M, N, \ldots = 0, 1, \ldots, 9$ and we use the following notations

$$
\begin{aligned}
F_{M_1\ldots M_5} &= 5\partial_{[M_1}A_{M_2\ldots M_5]} + 15(B_{[M_1M_2}\partial_{M_3}C_{M_4M_5]} - C_{[M_1M_2}\partial_{M_3}B_{M_4M_5]})\,; \\
\mathcal{F} &= F - F^*
\end{aligned}
$$

and all antisymmetrizations are with "weight" 1, e.g. $3\partial_{[M}B_{NK]}{=}\partial_M B_{NK}{-}\partial_N B_{MK}{-}\partial_K B_{NM}$. The dual forms are defined as

$$
\begin{aligned}
\varepsilon_{01\ldots 9} &= \sqrt{-g}\,, \\
\varepsilon^{01\ldots 9} &= -\frac{1}{\sqrt{-g}}\,, \\
\varepsilon^{M_1\ldots M_{10}} &= g^{M_1N_1}\cdots g^{M_{10}N_{10}}\varepsilon_{N_1\ldots N_{10}}\,, \\
(F^*)_{M_1\ldots M_k} &= 6\frac{1}{k!}\varepsilon_{M_1\ldots M_{10}}F^{M_{k+1}\ldots M_{10}} = \frac{1}{k!}\varepsilon^{N_1\ldots N_{10}}g_{M_1N_1}\cdots g_{M_kN_k}F_{N_{k+1}\ldots N_{10}}\,.
\end{aligned}
$$

$S(\psi)$ denotes the part of the action that depends on fermions. Since the quadratic action for fermions can be easily restored by using their equations of motion, we will discuss in this section only the bosonic part.

One can easily verify that (2.1) possesses all the gauge and global symmetries of the conventional covariant equations of motion. However, the equations of motion that follow from (2.1) differ from the conventional ones in many aspects. In particular, the 4-form potential now satisfies a second-order differential equation.

The general covariance of the action is achieved by introducing an auxiliary scalar field a. As was shown in [4, 5], there is a gauge symmetry of (2.1) that allows one to set $a = x_0$. Under this choice the action (2.1) does not depend on the components of the 4-form potential of the form $A^0{}_{MNP}$.

By using the units in which the radius of S^5 is set to be unity, the $AdS_5 \times S^5$ background solution can be written as

$$
\begin{aligned}
B_2 &= C_2 = \varphi = \chi = 0\,, \\
ds^2 &= \frac{1}{x_0^2}(dx_0^2 + \eta_{ij}dx^i dx^j) + d\Omega_5^2 = \dot{g}_{MN}dx^M dx^N\,,
\end{aligned}
$$

$$
\begin{aligned}
R_{abcd} &= -\dot{g}_{ac}\dot{g}_{bd} + \dot{g}_{ad}\dot{g}_{bc}\,, \qquad R_{ab} = -4\dot{g}_{ab}\,,\\
R_{\alpha\beta\gamma\delta} &= \dot{g}_{\alpha\gamma}\dot{g}_{\beta\delta} - \dot{g}_{\alpha\delta}\dot{g}_{\beta\gamma}\,, \qquad R_{\alpha\beta} = 4\dot{g}_{\alpha\beta}\,,\\
\bar{F}_{abcde} &= \varepsilon_{abcde}\,, \qquad \bar{F}_{\alpha\beta\gamma\delta\varepsilon} = \varepsilon_{\alpha\beta\gamma\delta\varepsilon}\,,
\end{aligned} \tag{2.2}
$$

where $a, b, c, \ldots$ and $\alpha, \beta, \gamma, \ldots$ are the AdS and the sphere indices respectively and η_{ij} is the 4-dimensional Minkowski metric. Representing the gravitational field and the 4-form potential as

$$
g_{MN} = \dot{g}_{MN} + h_{MN}\,, \qquad A_{MNPQ} = \dot{A}_{MNPQ} + a_{MNPQ}\,,
$$

decomposing (2.1) up to the second order and omitting full-derivative terms, one obtains the quadratic action[2]

$$
\begin{aligned}
S = \int d^{10}x\sqrt{-g}\Bigg(&-\frac{1}{4}\nabla_K h_{MN}\nabla^K h^{MN} + \frac{1}{2}\nabla_M h_{KN}\nabla^K h^{MN} - \frac{1}{2}\nabla_N h_K^K \nabla_M h^{MN} +\\
&+ \frac{1}{4}\nabla_M h_K^K \nabla^M h_N^N - \frac{2}{3!} h_N^M h_L^K \bar{F}_{MKM_1M_2M_3}\bar{F}^{NLM_1M_2M_3} -\\
&- \frac{4}{5!} f_{M_1\ldots M_5} f^{M_1\ldots M_5} - \frac{8}{5!} f_{M_1\ldots M_5} T^{M_1\ldots M_5} +\\
&+ \frac{1}{3!}\frac{\partial^L a \partial_K a}{(\partial a)^2}(\mathcal{F}+T)_{LM_1\ldots M_4}(\mathcal{F}+T)^{KM_1\ldots M_4} +\\
&+ \frac{4}{5!}\varepsilon^{M_1\ldots M_{10}}\bar{F}_{M_1\ldots M_5} B_{M_6M_7}\partial_{M_8} C_{M_9M_{10}} - 3(\partial_{[M}B_{NK]})^2 -\\
&- 3(\partial_{[M}C_{NK]})^2 - \frac{1}{2}(\partial_M\varphi)^2 - \frac{1}{2}(\partial_M\chi)^2\Bigg) + S(\psi)\,,
\end{aligned} \tag{2.3}
$$

where

$$
\begin{aligned}
f_{M_1\ldots M_5} &= 5\partial_{[M_1} a_{M_2\ldots M_5]}\,, \qquad \mathcal{F} = f - f^*\,,\\
T_{M_1\ldots M_5} &= \frac{1}{2} h_K^K \bar{F}_{M_1\ldots M_5} - 5 h^K_{[M_1}\bar{F}_{M_2\ldots M_5]K}\,.
\end{aligned}
$$

One can easily check that the 5-form T is antiself-dual.

The gauge symmetry of (2.3) (and of (2.1)) allows one to impose the following gauge conditions:

$$
\begin{aligned}
\nabla^\alpha h_{a\alpha} &= 0 = \nabla^\alpha h_{(\alpha\beta)}\,, \qquad h_{(\alpha\beta)} \equiv h_{\alpha\beta} - \frac{1}{5}\dot{g}_{\alpha\beta} h_\gamma^\gamma\,,\\
\nabla^\alpha a_{M_1M_2M_3\alpha} &= 0\,,\\
\nabla^\alpha B_{M\alpha} &= 0 = \nabla^\alpha C_{M\alpha}\,,
\end{aligned} \tag{2.4}
$$

This gauge choice does not remove all the gauge symmetry of the theory, for a detailed discussion of the residual symmetry see [28].

We begin our study of the action with the most difficult part, describing the gravitational fields and the 4-form potential.

[2] In what follows we omit the common factor $1/2\kappa^2$ in front of the action.

3. Gravitational fields and the 4-form potential

In what follows it is convenient to make the change of variable $x_0 \to \mathrm{e}^{x_0}$. Then the component $\dot{g}_{00}$ of the background metric is equal to unity. In the gauge $a = x_0$ the quadratic action describing the gravitational fields and the 4-form potential can be rewritten in the form

$$S(h,a) = S(h) + S(a) \tag{3.1}$$

$$\begin{aligned} S(h) = \int d^{10}x\sqrt{-g}\Big(&-\frac{1}{4}\nabla_K h_{MN}\nabla^K h^{MN} + \frac{1}{2}\nabla_M h_{KN}\nabla^K h^{MN} - \\ &- \frac{1}{2}\nabla_N h_K^K \nabla_M h^{MN} + \frac{1}{4}\nabla_M h_K^K \nabla^M h_N^N - \\ &- \frac{2}{3!}h_N^M h_L^K \bar{F}_{MKM_1M_2M_3}\bar{F}^{NLM_1M_2M_3} + \frac{1}{3!}T_{0\mu_1\ldots\mu_4}T^{0\mu_1\ldots\mu_4}\Big) \end{aligned} \tag{3.2}$$

$$S(a) = \int d^{10}x\sqrt{-g}\left(-\frac{4}{5!}f_{M_1\ldots M_5}f^{M_1\ldots M_5} - \frac{16}{5!}f_{\mu_1\ldots\mu_5}T^{\mu_1\ldots\mu_5} + \frac{1}{3!}\mathcal{F}_{0\mu_1\ldots\mu_4}\mathcal{F}^{0\mu_1\ldots\mu_4}\right), \tag{3.3}$$

where $\mu = 1, 2, \ldots, 9$.

Consider first the part depending on the 4-form potential. Taking into account that the nonvanishing components of T are given by

$$\begin{aligned} T_{a_1\ldots a_5} &= -\frac{1}{2}(h_a^a - h_\alpha^\alpha)\varepsilon_{a_1\ldots a_5}\,, \qquad & T_{\alpha a_1\ldots a_4} &= -h_\alpha^b \varepsilon_{ba_1\ldots a_4}\,, \\ T_{\alpha_1\ldots\alpha_5} &= -\frac{1}{2}(h_a^a - h_\alpha^\alpha)\varepsilon_{\alpha_1\ldots\alpha_5}\,, \qquad & T_{a\alpha_1\ldots\alpha_4} &= -h_a^\beta \varepsilon_{\beta\alpha_1\ldots\alpha_4}\,, \end{aligned} \tag{3.4}$$

we rewrite the action (3.3) in the form

$$\begin{aligned} S(a) = \int d^{10}x\sqrt{-g}\Big(&-\frac{1}{2(3!)^2}\varepsilon^{0\mu_1\ldots\mu_9}\partial_0 a_{\mu_1\ldots\mu_4}\partial_{\mu_5}a_{\mu_6\ldots\mu_9} - \frac{5}{3}\partial_{[\mu_1}a_{\mu_2\ldots\mu_5]}\partial^{[\mu_1}a^{\mu_2\ldots\mu_5]} + \\ &+ \frac{2}{3}\varepsilon^{0ijkl}h_0^\alpha(\partial_\alpha a_{ijkl} - 4\partial_l a_{ijk\alpha}) - \frac{1}{3}(h_a^a - h_\alpha^\alpha)\varepsilon^{\alpha_1\ldots\alpha_5}\partial_{\alpha_5}a_{\alpha_1\ldots\alpha_4} + \\ &+ \frac{2}{3}\varepsilon^{\alpha\beta\gamma\delta\varepsilon}h_\varepsilon^i(\partial_i a_{\alpha\beta\gamma\delta} - 4\partial_\alpha a_{i\beta\gamma\delta})\Big)\,. \end{aligned} \tag{3.5}$$

As was shown in [28], the gauge condition (2.4) implies that the components of the 4-form potential of the form $a_{i\alpha\beta\gamma}$ and $a_{\alpha\beta\gamma\delta}$ can be represented as follows:

$$a_{i\alpha\beta\gamma} = \varepsilon_{\alpha\beta\gamma}{}^{\delta\varepsilon}\nabla_\delta\phi_{i\varepsilon}\,, \qquad a_{\alpha\beta\gamma\delta} = \varepsilon_{\alpha\beta\gamma\delta}{}^{\varepsilon}\nabla_\varepsilon b\,. \tag{3.6}$$

It is also convenient to introduce the dual 1- and 0-forms for $a_{ijk\alpha}$ and a_{ijkl}:

$$a_{ijk\alpha} = \varepsilon_{ijk}{}^{l}a_{l\alpha}\,, \qquad a_{ijkl} = \varepsilon_{ijkl}a\,. \tag{3.7}$$

Then by using (3.6) and (3.7) and the gauge conditions (2.4) and (2.4), one gets the following expression for the action (3.5):

$$S(a)=\int d^{10}x\sqrt{-g}\Big(-16\partial_0 b\nabla_\alpha^2 a+8\nabla_\alpha a\nabla^\alpha a+8\nabla_i\nabla_\alpha^2 b\nabla^i b-8\nabla_\alpha^2 b\nabla_\beta^2 b-$$
$$-8(h_a^a-h_\alpha^\alpha)\nabla_\beta^2 b+16\partial_0\phi_i^\alpha(\nabla_\beta^2-4)a_\alpha^i-8a^{i\alpha}(\nabla_\beta^2-4)a_{i\alpha}+$$
$$+8\nabla_i a_\alpha^i\nabla_j a^{j\alpha}+16h_0^\alpha\nabla_i a_\alpha^i+8\nabla_i\phi_{j\alpha}(\nabla_\beta^2-4)(\nabla^i\phi^{j\alpha}-\nabla^j\phi^{i\alpha})-$$
$$-8(\nabla_\beta^2-4)\phi_{i\alpha}(\nabla_\gamma^2-4)\phi^{i\alpha}+16h_\alpha^i(\nabla_\beta^2-4)\phi_i^\alpha-$$
$$-\frac{1}{2}\varepsilon^{i_1\dots i_4}\varepsilon^{\alpha_1\dots\alpha_5}\partial_0 a_{i_1i_2\alpha_1\alpha_2}\partial_{\alpha_3}a_{i_3i_4\alpha_4\alpha_5}-$$
$$-2\nabla_i a_{jk\alpha\beta}(\nabla^i a^{jk\alpha\beta}-2\nabla^j a^{ik\alpha\beta})+2a_{ij\alpha\beta}(\nabla_\gamma^2-6)a^{ij\alpha\beta}\Big). \quad (3.8)$$

We see that (3.8) is a sum of actions for the scalar fields, the vector fields and the antisymmetric tensor fields on AdS_5.

The action for the antisymmetric fields will be treated first. Although the action is not manifestly covariant with respect to the isometry group of AdS_5, one achieves the covariance by introducing the additional fields $a_{0i\alpha\beta}$ that obey the follo wing equations:

$$a_{0i\alpha\beta}=\frac{1}{4}(-\nabla_\rho^2+6)^{-1}\varepsilon_{ijkl}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\nabla^\gamma\nabla^j a^{kl\delta\varepsilon}\,.$$

Then the action for the antisymmetric fields can be rewritten in the equivalent and manifestly covariant form:

$$S=\int d^{10}x\sqrt{-g}\left(-\frac{1}{2}\varepsilon^{a_1\dots a_5}\varepsilon^{\alpha_1\dots\alpha_5}\partial_{a_5}a_{a_1a_2\alpha_1\alpha_2}\partial_{\alpha_3}a_{a_3a_4\alpha_4\alpha_5}+2a_{ab\alpha\beta}(\nabla_\gamma^2-6)a^{ab\alpha\beta}\right). \quad (3.9)$$

Now expanding the fields $a_{ab\alpha\beta}$ in terms of the spherical harmonics

$$a_{ab\alpha\beta}=\frac{1}{2}a^+_{ab\alpha\beta}+\frac{1}{2}a^-_{ab\alpha\beta}\,,\qquad a^\pm_{ab\alpha\beta}(x,y)=\sum b_{ab}^{I_{10},\pm}(x)Y_{[\alpha\beta]}^{I_{10},\pm}(y)\,,$$

where the harmonics are eigenfunctions of the operator

$$({}^*\nabla)Y_{[\alpha\beta]}\equiv\varepsilon_{\alpha\beta}{}^{\gamma\delta\varepsilon}\nabla_\gamma Y_{[\delta\varepsilon]}\,,\qquad({}^*\nabla)Y^{k,\pm}_{[\alpha\beta]}=\pm2i(k+2)Y^{k,\pm}_{[\alpha\beta]}\,,\qquad k\geq0\,,$$

we rewrite action (3.9) as follows

$$S(b_{ab})=-\int_{AdS_5}d^5x\sqrt{-g_a}\sum(k+2)\left(\frac{i}{2}\varepsilon^{a_1\dots a_5}b^{k,+}_{a_1a_2}\partial_{a_3}b^{k,-}_{a_4a_5}+(k+2)b^{k,+}_{ab}b^{ab}_{k,-}\right). \quad (3.10)$$

Here and in what follows we suppose that the spherical harmonics of all types are orthonormal. We see that action (3.10) reproduces the part of the spectrum of [28, 29] for the antisymmetric fields.

This action cannot be used in the AdS/CFT correspondence because it's on-shell value vanishes. As follows from [19], one has to add to (3.10) the boundary term

$$I(b_{ab})=\int_{\partial AdS_5}d^4x\sqrt{-\bar{g}}\sum\frac{k+2}{2}b^{k,+}_{ij}b^{ij}_{k,-}\,, \quad (3.11)$$

where the indices i, j are contracted with the help of the metric induced on the boundary of AdS_5, and $\bar{g}$ is the determinant of the induced metric. Then the sum of the bulk action (3.10) and the boundary term (3.11) is the action that leads to the conformally-invariant two-point Green functions [19].

Now consider the part of (3.8) that depends on the scalar fields b and a. Eliminating the field a by using it's equation of motion one gets

$$S(b) = \int d^{10}x\sqrt{-g}\left(-8\nabla_a\nabla_\alpha b\nabla^a\nabla^\alpha b - 8\nabla^2_\alpha b\nabla^2_\beta b - 8(h^a_a - h^\alpha_\alpha)\nabla^2_\alpha b\right). \tag{3.12}$$

Note that the action is manifestly covariant.

To represent the action for the vector fields in a covariant form we introduce an auxiliary field $\varphi_{0\alpha}$ which obeys the equation $\varphi_{0\alpha} = \nabla_i a^i_\alpha$, and rewrite the action as follows

$$\begin{aligned} S(\phi) = \int d^{10}x\sqrt{-g}\,\Big(&16\partial_0\phi^\alpha_i(\nabla^2_\beta - 4)a^i_\alpha - 8a^{i\alpha}(\nabla^2_\beta - 4)a_{i\alpha} + 16\varphi_{0\alpha}\nabla_i a^{i\alpha} - \\ &- 8\varphi_{0\alpha}\varphi^\alpha_0 + 16h^\alpha_0\nabla_i a^i_\alpha + 8\nabla_i\phi_{j\alpha}(\nabla^2_\beta - 4)(\nabla^i\phi^{j\alpha} - \nabla^j\phi^{i\alpha}) - \\ &- 8(\nabla^2_\beta - 4)\phi_{i\alpha}(\nabla^2_\gamma - 4)\phi^{i\alpha} + 16h^i_\alpha(\nabla^2_\beta - 4)\phi^\alpha_i\Big). \end{aligned} \tag{3.13}$$

The field $a_{i\alpha}$ satisfies the following equation of motion

$$a_{i\alpha} = \partial_0\phi_{i\alpha} + (-\nabla^2_\beta + 4)^{-1}\nabla_i(\varphi_{0\alpha} + h_{0\alpha}).$$

Introducing the field $\phi_{0\alpha}$ by the formula $(\nabla^2_\beta - 4)\phi_{0\alpha} = \varphi_{0\alpha} + h_{0\alpha}$ and eliminating the field $a_{i\alpha}$ from (3.13), we get

$$\begin{aligned} S(\phi) = \int d^{10}x\sqrt{-g}\,\Big(&8\nabla_a\phi_{b\alpha}(\nabla^2_\beta - 4)(\nabla^a\phi^{b\alpha} - \nabla^b\phi^{a\alpha}) - 8(\nabla^2_\beta - 4)\phi_{a\alpha}(\nabla^2_\gamma - 4)\phi^{a\alpha} + \\ &+ 16h^a_\alpha(\nabla^2_\beta - 4)\phi^\alpha_a - 8h_{0\alpha}h^{0\alpha}\Big). \end{aligned} \tag{3.14}$$

Only the last term violates the manifest covariance of the action. However we will see in a moment that this term is cancelled by a term coming from the action for the gravitational fields.

To this end, by using the gauge conditions for the gravitational fields and eq. (3.4), we rewrite the action $S(h)$ as follows

$$\begin{aligned} S(h) = \int d^{10}x\sqrt{-g}\,\Big(&-\frac{1}{4}\nabla_K h_{ab}\nabla^K h^{ab} + \frac{1}{2}\nabla_a h^{ab}\nabla^c h_{cb} - \frac{1}{2}\nabla_a h^c_c\nabla_b h^{ba} + \\ &+ \frac{1}{4}\nabla_M h^a_a\nabla^M h^b_b + \frac{1}{2}h_{ab}h^{ab} + \frac{1}{2}(h^a_a)^2 - \frac{1}{2}\nabla_a h^\alpha_\alpha\nabla_b h^{ba} + \\ &+ \frac{1}{2}\nabla_b h^a_a\nabla^b h^\alpha_\alpha + \frac{2}{5}\nabla_\beta h^a_a\nabla^\beta h^\alpha_\alpha + 2h^a_a h^\alpha_\alpha + \frac{1}{5}\nabla_b h^\alpha_\alpha\nabla^b h^\beta_\beta + \\ &+ \frac{3}{25}\nabla_\gamma h^\alpha_\alpha\nabla^\gamma h^\beta_\beta - \frac{13}{5}(h^\alpha_\alpha)^2 - \frac{1}{4}\nabla_K h_{(\alpha\beta)}\nabla^K h^{(\alpha\beta)} - \\ &- \frac{1}{2}h_{(\alpha\beta)}h^{(\alpha\beta)} - \frac{1}{2}\nabla_a h_{b\alpha}(\nabla^a h^{b\alpha} - \nabla^b h^{a\alpha}) - \\ &- \frac{1}{2}\nabla_\beta h_{a\alpha}\nabla^\beta h^{a\alpha} - 6h^a_\alpha h^\alpha_a + 8h_{0\alpha}h^{0\alpha}\Big). \end{aligned} \tag{3.15}$$

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So, we see that although the gauge condition $a = x_0$ violates the manifest covariance of the quadratic action with respect to the action of the isometry group of $AdS_5 \times S^5$, one can restore it by introducing auxiliary fields.

There is no problem with the fields $h^{(\alpha\beta)}$. Being expanded into spherical harmonics they directly lead to the corresponding part of the spectrum of [28, 29].

Consider the action for the vector fields that is a sum of (3.14) and the last line of (3.15):

$$S(vect) = \int d^{10}x\sqrt{-g}\Big(8\ \nabla_a\phi_{b\alpha}(\nabla^2_\beta - 4)(\nabla^a\phi^{b\alpha} - \nabla^b\phi^{a\alpha}) - \\ - 8(\nabla^2_\beta - 4)\phi_{a\alpha}(\nabla^2_\gamma - 4)\phi^{a\alpha}16h^a_\alpha(\nabla^2_\beta - 4)\phi^\alpha_a - \\ - \frac{1}{2}\nabla_a h_{b\alpha}(\nabla^a h^{b\alpha} - \nabla^b h^{a\alpha}) - \frac{1}{2}\nabla_\beta h_{a\alpha}\nabla^\beta h^{a\alpha} - 6h^a_\alpha h^\alpha_a\Big)\,. \tag{3.16}$$

Expanding the fields into a set of the spherical harmonics as follows

$$h_{a\alpha}(x,y) = \sum B^{I_5}_a(x)Y^{I_5}_\alpha(y)\,, \qquad \phi_{a\alpha}(x,y) = \sum \phi^{I_5}_a(x)Y^{I_5}_\alpha(y)\,, \\ (\nabla^2_\alpha - 4)Y^k_\alpha = -(k+1)(k+3)Y^k_\alpha\,,$$

and making the change of variables [28]

$$A^k_a = B^k_a - 4(k+3)\phi^k_a\,, \qquad C^k_a = B^k_a + 4(k+1)\phi^k_a$$

we obtain the final action for the vector fields:

$$S(vect) = \int_{AdS_5} d^5x\sqrt{-g_a}\sum\Big(\frac{k+1}{2(k+2)}\Big(-\frac{1}{4}(F_{ab}(A^k))^2 - \frac{1}{2}(k^2-1)(A^k_a)^2\Big) + \\ + \frac{k+3}{2(k+2)}\Big(-\frac{1}{4}(F_{ab}(C^k))^2 - \frac{1}{2}(k+3)(k+5)(C^k_a)^2\Big)\Big)\,, \tag{3.17}$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_a$.

Now we proceed with the most complicated part of the action $S(h,a)$ which depends on the gravitational fields h_{ab} and the scalar fields h^α_α and b:

$$S(h_{ab},\pi,b) = \int d^{10}x\sqrt{-g}\Big(-\frac{1}{4}\nabla_K h_{ab}\nabla^K h^{ab} + \frac{1}{2}\nabla_a h^{ab}\nabla^c h_{cb} - \frac{1}{2}\nabla_a h^c_c\nabla_b h^{ba} + \\ + \frac{1}{4}\nabla_M h^a_a\nabla^M h^b_b + \frac{1}{2}h_{ab}h^{ab} + \frac{1}{2}(h^a_a)^2 - \\ - \frac{1}{2}\nabla_a\pi\nabla_b h^{ba} + \frac{1}{2}\nabla_b h^a_a\nabla^b\pi + \frac{2}{5}\nabla_\beta h^a_a\nabla^\beta\pi + 2h^a_a\pi + \\ + \frac{1}{5}\nabla_b\pi\nabla^b\pi + \frac{3}{25}\nabla_\gamma\pi\nabla^\gamma\pi - \frac{13}{5}\pi^2 - \\ - 8\nabla_a\nabla_\alpha b\nabla^a\nabla^\alpha b - 8\nabla^2_\alpha b\nabla^2_\beta b - 8(h^a_a - \pi)\nabla^2_\alpha b\Big)\,, \tag{3.18}$$

where we denote $h^\alpha_\alpha = \pi$, following [28].

First of all we need to remove the mixed terms of the form πh_{ab} and bh_{ab}, i.e. linear in h_{ab}. To this end we make the following shift of the gravitational fields:

$$h_{ab} = \varphi_{ab} + \frac{1}{5}\dot{g}_{ab}\eta + 2\nabla_a\nabla_b\zeta\,. \tag{3.19}$$

The requirement that there is no term linear in φ_{ab} fixes η and ζ to be

$$\zeta = (-\nabla_\alpha^2 + 3)^{-1}\left(\frac{1}{5}\pi - 6b\right), \qquad \eta = -10\zeta - 20b - \pi\,. \tag{3.20}$$

Then after straightforward but cumbersome calculations the action (3.18) is found to be

$$\begin{aligned}
S(h_{ab},\pi,b) &= S(\varphi_{ab}) + S(\pi,b),\\
S(\varphi_{ab}) &= \int d^{10}x\sqrt{-g}\left(-\frac{1}{4}\nabla_K\varphi_{ab}\nabla^K\varphi^{ab} + \frac{1}{2}\nabla_a\varphi^{ab}\nabla^c\varphi_{cb} - \frac{1}{2}\nabla_a\varphi_c^c\nabla_b\varphi^{ba} + \right.\\
&\qquad \left. + \frac{1}{4}\nabla_M\varphi_a^a\nabla^M\varphi_b^b + \frac{1}{2}\varphi_{ab}\varphi^{ab} + \frac{1}{2}(\varphi_a^a)^2\right) \qquad (3.21)\\
S(\pi,b) &= \int d^{10}x\sqrt{-g}\Bigg(4(-\nabla_\alpha^2+3)^{-1}\left(\frac{1}{5}\pi - 6b\right)(\nabla_a^2 - 5)\left(\frac{1}{5}\pi - 6b\right) + \\
&\qquad + 48(\nabla_a b)^2 - 80(\nabla_\alpha b)^2 - \frac{2}{25}(\nabla_a\pi)^2 - \frac{2}{25}(\nabla_\alpha\pi)^2 - \\
&\qquad - \frac{16}{5}\nabla_a b\nabla^a\pi - 16\nabla_\alpha b\nabla^\alpha\pi + 240b^2 - 4\pi^2 - 16b\pi - \\
&\qquad - 8\nabla_a\nabla_\alpha b\nabla^a\nabla^\alpha b - 8\nabla_\alpha^2 b\nabla_\beta^2 b\Bigg) \qquad (3.22)
\end{aligned}$$

The absence of higher-derivative terms in (3.22) is explained by the general covariance of (2.1).

Now to get the final action for the scalar fields we expand them in the spherical harmonics as

$$\pi(x,y) = \sum \pi^{I_1}(x)Y^{I_1}(y)\,, \qquad b(x,y) = \sum b^{I_1}(x)Y^{I_1}(y)\,, \qquad \nabla_\alpha^2 Y^k = -k(k+4)Y^k\,,$$

and make the redefinition of the fields [18]

$$\pi_k = 10ks_k + 10(k+4)t_k\,, \qquad b_k = -s_k + t_k\,.$$

Then, after some algebra we obtain the action

$$\begin{aligned}
S(s,t) = \int_{AdS_5} d^5x\sqrt{-g_a}\sum\Bigg(&\frac{32k(k-1)(k+2)}{k+1}\left(-\frac{1}{2}\nabla_a s_k\nabla^a s_k - \frac{1}{2}k(k-4)s_k^2\right) + \\
&+ \frac{32(k+2)(k+4)(k+5)}{k+3}\times \\
&\times\left(-\frac{1}{2}\nabla_a t_k\nabla^a t_k - \frac{1}{2}(k+4)(k+8)t_k^2\right)\Bigg)\,.
\end{aligned} \tag{3.23}$$

Note that the action for the scalar fields s_k coincides with the one found in [18]. The fact that (3.23) does not depend on s_0 and s_1 means that these modes are gauge.

Finally we discuss the action (3.21) for the gravitational fields. Expanding the fields in a set of spherical harmonics, we see that the zero mode describes a massless graviton on AdS_5. We need to show that the massive modes describe traceless symmetric tensor fields. This can be done in two ways. First of all one can use the equation of motion[3] $h_a^a + \frac{3}{5}\pi = 0$ that enters the complete set of equations of motion. Then by using the equations of motion for b and π one can easily show that φ_a^a vanishes on shell. The second way is to decouple φ_a^a from the traceless part of φ_{ab}. To this end we make the following change of variables:

$$\varphi_{ab} = \phi_{(ab)} + \frac{1}{5}\dot{g}_{ab}\phi - \frac{3}{5}\nabla_a\nabla_b\nabla_\alpha^{-2}\phi\,, \tag{3.24}$$

$$\varphi_a^a = \phi - \frac{3}{5}\nabla_a^2\nabla_\alpha^{-2}\phi\,,, \tag{3.25}$$

where $\phi_{(ab)}$ is a traceless symmetric tensor. Then the action (3.21) acquires the form

$$S(\varphi_{ab}) = S(\phi_{(ab)}) + S(\phi)\,,$$

$$S(\phi_{(ab)}) = \int d^{10}x\sqrt{-g}\left(-\frac{1}{4}\nabla_K\phi_{(ab)}\nabla^K\phi^{(ab)} + \frac{1}{2}\nabla_a\phi^{(ab)}\nabla^c\phi_{(cb)} + \frac{1}{2}\phi_{(ab)}\phi^{(ab)}\right), \tag{3.26}$$

$$S(\phi) = \int d^{10}x\sqrt{-g}\left(\frac{1}{5}(\phi - \frac{3}{5}\nabla_a^2\nabla_\alpha^{-2}\phi)(-\nabla_\alpha^2 + 3)\phi\right), \tag{3.27}$$

So, we see that (3.26) is the action for the traceless massive symmetric tensor field that leads to the same equations of motion and the same spectrum as was obtained in [28, 29].

It is obvious that the action (3.27) leads to the equation

$$\phi - \frac{3}{5}\nabla_a^2\nabla_\alpha^{-2}\phi = 0 = \varphi_a^a\,. \tag{3.28}$$

Thus we again conclude that φ_a^a vanishes on shell. Although the field ϕ satisfies the second-order equation it does not describe any dynamical mode. The reason is that to make the transformation (3.25) well-defined we have to impose a certain boundary condition on the field ϕ, ensuring the invertibility of the operator $1 - \frac{3}{5}\nabla_a^2\nabla_\alpha^{-2}$. Then from (3.28) we get that ϕ always vanishes on shell.

Thus, we have completed the discussion of the gravitational fields and the 4-form potential, and now we proceed with the antisymmetric fields B and C.

4. Antisymmetric fields

The action for the antisymmetric tensor fields extracted from (2.3) is given by

$$\begin{aligned} S = \int d^{10}x\sqrt{-g}\Big(&-\nabla_M B_{NK}(\nabla^M B^{NK} - \nabla^N B^{MK} - \nabla^K B^{NM}) - \\ &- \nabla_M C_{NK}(\nabla^M C^{NK} - \nabla^N C^{MK} - \nabla^K C^{NM}) - \\ &- 4\varepsilon^{\alpha\beta\gamma\delta\varepsilon}B_{\alpha\beta}\partial_\gamma C_{\delta\varepsilon} - 4\varepsilon^{abcde}B_{ab}\partial_c C_{de}\Big)\,. \end{aligned} \tag{4.1}$$

[3] This equation is valid only for the $k \geq 2$ modes. One can apply the remaining conformal diffeomorphism to obtain $h_a^a = -\frac{3}{5}\pi$ [28].

Although the equations of motion obtained from the action coincides with the ones from [28], they are not diagonal, and the fields B and C do not correspond to primary operators of the $\mathcal{N}=4$ SYM model. Therefore, the main purpose of this section is to introduce a proper set of fields and to rewrite the action in terms of the fields. To this end it is convenient to replace the two real fields by one complex field:

$$A=\sqrt{2}(B+iC)\,,\quad \bar{A}=\sqrt{2}(B-iC)\,,\quad B=\frac{1}{2\sqrt{2}}(A+\bar{A})\,,\quad C=\frac{1}{2\sqrt{2}i}(A-\bar{A})\,.$$

Rewriting (4.1) in terms of A and $\bar{A}$, one gets, up to total derivative terms,

$$S=\int d^{10}x\sqrt{-g}\Big(-\frac{1}{2}\nabla_M\bar{A}_{NK}\left(\nabla^M A^{NK}-\nabla^N A^{MK}-\nabla^K A^{NM}\right)+ \\ +i\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\bar{A}_{\alpha\beta}\partial_\gamma A_{\delta\varepsilon}+i\varepsilon^{abcde}\bar{A}_{ab}\partial_c A_{de}\Big)\,. \tag{4.2}$$

Taking into account that the fields A_{MN} satisfy the gauge conditions $\nabla^\alpha A_{\alpha\beta}=\nabla^\alpha A_{\alpha b}=0$, one can rewrite action (4.2) as follows

$$S=\int d^{10}x\sqrt{-g}\Big(-\frac{1}{2}\left(\nabla_a\bar{A}_{\alpha\beta}\nabla^a A^{\alpha\beta}+\nabla_\gamma\bar{A}_{\alpha\beta}\nabla^\gamma A^{\alpha\beta}+6\bar{A}_{\alpha\beta}A^{\alpha\beta}\right)- \\ -\left(\nabla_a\bar{A}_{b\alpha}(\nabla^a A^{b\alpha}-\nabla^b A^{a\alpha})+\nabla_\beta\bar{A}_{a\alpha}\nabla^\beta A^{a\alpha}+4\bar{A}_{a\alpha}A^{a\alpha}\right)- \\ -\frac{1}{2}\left(\nabla_a\bar{A}_{bc}(\nabla^a A^{bc}-\nabla^b A^{ac}-\nabla^c A^{ba})+\nabla_\alpha\bar{A}_{ab}\nabla^\alpha A^{ab}\right)+ \\ +i\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\bar{A}_{\alpha\beta}\partial_\gamma A_{\delta\varepsilon}+i\varepsilon^{abcde}\bar{A}_{ab}\partial_c A_{de}\Big) \tag{4.3}$$

It is obvious that the first and second lines of (4.3) are just the actions for scalar and vector fields respectively on AdS_5. These actions directly lead to the spectrum found in [28, 29].

Let us consider the action describing the antisymmetric tensor fields on AdS_5:

$$S_{as}=\int d^{10}x\sqrt{-g}\Big(-\frac{1}{2}\left(\nabla_a\bar{A}_{bc}(\nabla^a A^{bc}-\nabla^b A^{ac}-\nabla^c A^{ba})+\nabla_\alpha\bar{A}_{ab}\nabla^\alpha A^{ab}\right)+ \\ +i\varepsilon^{abcde}\bar{A}_{ab}\partial_c A_{de}\Big) \tag{4.4}$$

Introducing the auxiliary fields P_{ab} and $\bar{P}_{ab}$, we rewrite the action in the first-order formalism:

$$S_{as}=\int d^{10}x\sqrt{-g}\left(-\frac{i}{2}\varepsilon^{abcde}\bar{P}_{ab}\partial_c A_{de}+\frac{i}{2}\varepsilon^{abcde}P_{ab}\partial_c\bar{A}_{de}- \right. \\ \left. -2\bar{P}_{ab}P^{ab}-\frac{1}{2}\nabla_\alpha\bar{A}_{ab}\nabla^\alpha A^{ab}+i\varepsilon^{abcde}\bar{A}_{ab}\partial_c A_{de}\right) \tag{4.5}$$

Changing the variables

$$
\begin{aligned}
A_1 &= \frac{1}{2}(-\nabla_\alpha\nabla^\alpha + 4)^{1/4}A + (-\nabla_\alpha\nabla^\alpha + 4)^{-1/4}(P - A)\\
A_2 &= \frac{1}{2}(-\nabla_\alpha\nabla^\alpha + 4)^{1/4}\bar{A} - (-\nabla_\alpha\nabla^\alpha + 4)^{-1/4}(\bar{P} - \bar{A})
\end{aligned}
$$

one gets the following action

$$
\begin{aligned}
S_{as} = -\int d^{10}x\sqrt{-g}\Bigg(&\frac{i}{2}\varepsilon^{abcde}\left(\bar{A}_{1ab}\partial_c A_{1de} + \bar{A}_{2ab}\partial_c A_{2de}\right) + \\
&+\left(\sqrt{(-\nabla_\alpha\nabla^\alpha + 4)} + 2\right)\bar{A}_{1ab}A_1^{ab} + \\
&+\left(\sqrt{(-\nabla_\alpha\nabla^\alpha + 4)} - 2\right)\bar{A}_{2ab}A_2^{ab}\Bigg)\,. \qquad (4.6)
\end{aligned}
$$

Taking into account that the eigenvalues of the operator $\sqrt{(-\nabla_\alpha\nabla^\alpha + 4)}$ are equal to $k+2;\ k \geq 0$, one obtains the spectrum of [28, 29]. This action has the same form as the action (3.10) for the antisymmetric fields coming from the 4-form potential, and, therefore, one has to add to the action the following boundary term

$$
I_{as} = \int_{\partial AdS_5\times S_5} d^9x\sqrt{-\bar{g}}\left(\frac{1}{2}\bar{A}_{1ij}A_1^{ij} + \frac{1}{2}\bar{A}_{2ij}A_2^{ij}\right). \qquad (4.7)
$$

There is no need to add boundary terms to the actions obtained in the previous section for the scalar, vector and symmetric tensor fields. The actions can be directly used for computing two-point Green functions in the framework of the AdS/CFT correspondence. Thus we have completed the discussion of the quadratic action for bosonic fields of type IIB supergravity, and now we proceed with the analysis of the fermions.

5. Fermion fields

We begin the consideration of the fermion fields with the simplest spinor case. The action for the spinor field that leads to the covariant equations of motion [1, 2, 3, 28] has the form

$$
S = \int d^{10}x\sqrt{-g}\left(\hat{\bar{\lambda}}\Gamma^M D_M\hat{\lambda} - \frac{i}{2\cdot 5!}\hat{\bar{\lambda}}\Gamma^{M_1\cdots M_5}F_{M_1\cdots M_5}\hat{\lambda}\right). \qquad (5.1)
$$

Here $\hat{\bar{\lambda}} = i\lambda^*\Gamma^{\hat{1}}$ and D_M is a covariant derivative. Recall that $M, N, P, \ldots,\ a, b, c, \ldots$ and $\alpha, \beta, \gamma, \ldots$ are curved ten-dimensional, AdS_5 and S_5 indices respectively. We denote $\hat{M}, \hat{N}, \hat{P}, \ldots,\ \hat{a}, \hat{b}, \hat{c}, \ldots,\ \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \ldots$ the corresponding flat indices. We choose the following representation of the Γ-matrices

$$
\begin{gathered}
\Gamma^a = \sigma^1\otimes I_4\otimes\gamma^a\,, \qquad \Gamma^\alpha = -\sigma^2\otimes\tau^\alpha\otimes I_4\,,\\
\{\Gamma_{\hat{M}}, \Gamma_{\hat{N}}\} = 2\eta_{\hat{M}\hat{N}}\,, \qquad \{\gamma_{\hat{a}}, \gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}}\,, \qquad \{\tau_{\hat{\alpha}}, \tau_{\hat{\beta}}\} = 2\delta_{\hat{\alpha}\hat{\beta}}\,.
\end{gathered}
$$

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In this representation the matrix Γ_{11} is equal to

$$\Gamma_{11} = \Gamma^{\hat{0}} \cdots \Gamma^{\hat{9}} = \begin{pmatrix} I_{16} & 0 \\ 0 & -I_{16} \end{pmatrix}.$$

Since the spinor is right-handed, it has the form

$$\hat{\lambda} = \frac{1}{2}(1 - \Gamma_{11})\hat{\lambda} = \begin{pmatrix} 0 \\ \lambda \end{pmatrix}.$$

Taking into account that

$$\gamma^{\hat{0}} \cdots \gamma^{\hat{4}} = i \cdot I_4 \,, \qquad \tau^{\hat{5}} \cdots \tau^{\hat{9}} = I_4$$

we find

$$\frac{i}{2 \cdot 5!} \Gamma^{M_1 \cdots M_5} F_{M_1 \cdots M_5} = - \begin{pmatrix} 0 & I_{16} \\ 0 & 0 \end{pmatrix} = -\sigma^+ \otimes I_{16} \tag{5.2}$$

and rewrite (5.1) in the form

$$S = \int d^{10}x \sqrt{-g}\, \bar{\lambda} \left(\gamma^a D_a + i\tau^\alpha D_\alpha + 1 \right) \lambda \,. \tag{5.3}$$

Here and in what follows we use the same notation γ^a (τ^α) for the 4×4 matrices and for the 16×16 matrices $I_4 \otimes \gamma^a$ $(\tau^\alpha \otimes I_4)$. Expanding the spinor in a set of the spherical harmonics

$$\lambda(x,y) = \sum_{k \geq 0} \left(\lambda_k^+(x) \Xi_k^+(y) + \lambda_k^-(x) \Xi_k^-(y) \right),$$
$$\tau^\alpha D_\alpha \Xi_k^\pm = m^{I_L} \Xi_k^\pm = \mp i \left(k + \frac{5}{2} \right) \Xi_k^\pm \,,$$

we obtain the action for the spinor fields

$$S = \int d^5x \sqrt{-g_a} \sum_{k \geq 0} \left(\bar{\lambda}_k^+ \left(\gamma^a D_a + k + \frac{7}{2} \right) \lambda_k^+ + \bar{\lambda}_k^- \left(\gamma^a D_a - k - \frac{3}{2} \right) \lambda_k^- \right). \tag{5.4}$$

It is obvious that the action vanishes on shell. Therefore, in the framework of the AdS/CFT correspondence one has to add a boundary term to (5.4). This boundary term

$$I = \frac{1}{2} \int d^4x \sqrt{-\bar{g}} \sum_{k \geq 0} \left(-\bar{\lambda}_k^+ \lambda_k^+ + \bar{\lambda}_k^- \lambda_k^- \right) \tag{5.5}$$

was found in [11] up to a numerical factor, and the factor was fixed in [9] by using the hamiltonian formulation of the action.

Now we proceed with the gravitino field. The action for the gravitino field leading to the covariant equations of motion has the form

$$S = \int d^{10}x \sqrt{-g} \left(\bar{\hat{\psi}}_M \Gamma^{MNP} D_N \hat{\psi}_P + \frac{i}{4 \cdot 5!} \bar{\hat{\psi}}_M \Gamma^{MNP} \Gamma^{M_1 \cdots M_5} F_{M_1 \cdots M_5} \Gamma_N \hat{\psi}_P \right), \tag{5.6}$$

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Taking into account that the gravitino field is left-handed

$$\hat{\psi}_M = \frac{1}{2}(1+\Gamma_{11})\hat{\psi}_M = \begin{pmatrix} \psi_M \\ 0 \end{pmatrix},$$

we rewrite (5.6) as follows

$$\begin{aligned} S = \int d^{10}x\sqrt{-g}\Big(\bar{\psi}_a\Big(&\gamma^{abc}D_b\psi_c - i\gamma^{ab}\tau^\alpha D_b\psi_\alpha + \\ &+ i\gamma^{ab}\tau^\alpha D_\alpha\psi_b + \gamma^a\tau^{\alpha\beta}D_\alpha\psi_\beta - \gamma^{ab}\psi_b\Big) + \\ &+ \bar{\psi}_\alpha\Big(-i\tau^{\alpha\beta\gamma}D_\beta\psi_\gamma - i\gamma^{ab}\tau^\alpha D_a\psi_b + \\ &+ \gamma^a\tau^{\alpha\beta}D_\beta\psi_a - \gamma^a\tau^{\alpha\beta}D_a\psi_\beta + \tau^{\alpha\beta}\psi_\beta\Big)\Big). \end{aligned} \tag{5.7}$$

As was shown in [28], by using local supersymmetries, one can choose the gravitino field to be of the form

$$\psi_\alpha = \psi_{(\alpha)} + D_{(\alpha)}\psi + \tau_\alpha\eta^+ ,$$

where $\tau^\alpha\psi_{(\alpha)} = \tau^\alpha D_{(\alpha)}\psi = D^\alpha\psi_{(\alpha)} = 0$, and η^+ is the Killing spinor, which obeys $(D_\alpha + \frac{i}{2}\tau_\alpha)\eta^+ = 0$. Then (5.7) acquires the form

$$S = \int d^{10}x\sqrt{-g}\Big(\bar{\psi}^{(\alpha)}\left(\gamma^a D_a\psi_{(\alpha)} - \psi_{(\alpha)} - i\tau^\beta D_\beta\psi_{(\alpha)}\right) + \tag{5.8}$$

$$+\bar{\psi}_a\left(\gamma^{abc}D_b\psi_c + i\gamma^{ab}\tau^\alpha D_\alpha\psi_b - \gamma^{ab}\psi_b - \gamma^a\left(\frac{4}{5}\hat{D}^2\psi + 5\psi\right)\right) +$$

$$+\bar{\psi}\tilde{D}_{(\alpha)}\left(-\gamma^a D_a D^{(\alpha)}\psi - \gamma^a\tau^{\alpha\beta}D_\beta\psi_a + D^{(\alpha)}\psi + i\hat{D}D^{(\alpha)}\psi\right) + \tag{5.9}$$

$$+\bar{\psi}^+_a\left(\gamma^{abc}D_b\psi^+_c + \frac{3}{2}\gamma^{ab}\psi^+_b - 5i\gamma^{ab}D_b\eta^+ - 10i\gamma^a\eta^+\right) +$$

$$+\bar{\eta}^+\left(-10\eta^+ - 20\gamma^a D_a\eta^+ - 5i\gamma^{ab}D_a\psi^+_b - 10i\gamma^a\psi^+_a\right)\Big). \tag{5.10}$$

Here $\hat{D} \equiv \tau^\alpha D_\alpha$, $\tilde{D}_{(\alpha)} \equiv D_\alpha - \frac{1}{5}\hat{D}\tau_\alpha$, ψ^+_a denotes the lowest mode of the gravitino field that is proportional to a Killing spinor, and ψ_a does not contain ψ^+_a in its expansion into harmonics.

Expanding the spinor $\psi_{(\alpha)}$ into a set of the spherical harmonics

$$\begin{aligned} &\psi_{(\alpha)}(x,y) = \sum \psi^{I_T}(x)\Xi^{I_T}_{(\alpha)}(y) \\ &\gamma^\beta D_\beta \Xi^{I_T}_{(\alpha)} = m^{I_T}\Xi^{I_T}_{(\alpha)} = \mp i\left(k+\frac{5}{2}\right)\Xi^{I_T}_{(\alpha)}, \qquad k\geq 1, \end{aligned}$$

we represent the action (5.8) for the spinor fields in the form

$$S = \int d^5x\sqrt{-g_a}\sum \bar{\psi}^{I_T}\left(\gamma^a D_a + im^{I_T} - 1\right)\psi^{I_T}. \tag{5.11}$$

One has to add to the action a boundary term of the form (5.5).

To show that the action (5.9) leads to the spectrum obtained in [28], we expand the gravitino field ψ_a and the spinor field ψ into harmonics and rewrite (5.9) as follows

$$S = \int d^5x\sqrt{-g_a}\sum\Bigg(\bar\psi_a^{I_L}\left(\gamma^{abc}D_b\psi_c^{I_L} + (im^{I_L}-1)\gamma^{ab}\psi_b^{I_L} - \left(\frac{4}{5}m_{I_L}^2+5\right)\gamma^a\psi^{I_L}\right) - \\ - \left(\frac{4}{5}m_{I_L}^2+5\right)\bar\psi^{I_L}\left(\gamma^a D_a\psi^{I_L} - \gamma^a\psi_a^{I_L} - \left(1+\frac{3i}{5}m^{I_L}\right)\psi^{I_L}\right)\Bigg) \tag{5.12}$$

Performing the shift of the gravitino fields

$$\psi_a^{I_L} \to \psi_a^{I_L} + D_a\chi_1^{I_L} + \gamma_a\chi_2^{I_L}$$

and requiring the decoupling of the fields from ψ^{I_L}, we find

$$\chi_1^{I_L} = \frac{3}{5}\cdot\frac{5+2im^{I_L}}{1+2im^{I_L}}\psi^{I_L}\,,$$
$$\chi_2^{I_L} = \frac{1}{5}\cdot\frac{(1-im^{I_L})(5+2im^{I_L})}{1+2im^{I_L}}\psi^{I_L}\,.$$

Then the action (5.12) acquires the form

$$S = \int d^5x\sqrt{-g_a}\sum\Bigg(\bar\psi_a^{I_L}\left(\gamma^{abc}D_b\psi_c^{I_L} + (im^{I_L}-1)\gamma^{ab}\psi_b^{I_L}\right) + \\ + \frac{2(5+2im^{I_L})(5-2im^{I_L})^2}{25(1+2im^{I_L})}\bar\psi^{I_L}\left(\gamma^a D_a\psi^{I_L} + (3+im^{I_L})\psi^{I_L}\right)\Bigg)\,. \tag{5.13}$$

The action (5.13) is the standard action for the Rarita-Schwinger field. To show that the mode $\gamma^a\psi_a$ is unphysical one can make the change of variables

$$\psi_a^{I_L} = \varphi_{(a)}^{I_L} + D_a\chi^{I_L} - \frac{1}{3}(im^{I_L}-1)\gamma_a\chi^{I_L}\,,$$
$$\gamma^a\psi_a^{I_L} = \gamma^a D_a\chi^{I_L} - \frac{5}{3}(im^{I_L}-1)\chi^{I_L}$$

and get the following expression for (5.13)

$$S = \int d^5x\sqrt{-g_a}\sum\Bigg(\bar\varphi_{I_L}^{(a)}\left(\gamma^b D_b\varphi_{(a)}^{I_L} - (im^{I_L}-1)\varphi_{(a)}^{I_L}\right) + \\ + \left(\frac{4}{3}(im^{I_L}-1)^2 - 3\right)\bar\chi^{I_L}\left(\gamma^a D_a\chi^{I_L} - \frac{5}{3}(im^{I_L}-1)\chi^{I_L}\right)\Bigg)\,. \tag{5.14}$$

The same reasoning as after (3.28) leads to the conclusion that the spinor χ always vanishes on shell. We see that the kinetic term for the gravitino fields is just the sum of the kinetic terms for the spinor fields.[4] Then, one can easily show by using the results obtained in [20, 21] and the hamiltonian approach of [9], that the boundary term one has to add to (5.14) again has the form (5.5).

The consideration of the modes ψ_a^+ and the Killing spinor η^+ goes the same way. One makes the shift

$$\psi_a^+ \to \psi_a^+ + \frac{5}{3}i\eta^+$$

and arrives at the action (5.13) for the massless gravitino and the action

$$S = \int d^5x\sqrt{-g_a}\frac{40}{3}\bar{\eta}^+ \left(\gamma^a D_a \eta^+ - \frac{11}{2}\eta^+\right) \tag{5.15}$$

for the Killing spinor. This completes our derivation of the quadratic action for type IIB supergravity on $AdS_5 \times S^5$.

6. Conclusion

An important feature of the quadratic action we obtained in this paper is that, although we started from the noncovariant gauge condition, the final action possesses manifest invariance with respect to the isometry group of the $AdS_5 \times S^5$ background. It allows one to expect that there may exist a covariant description of the complete action for type IIB supergravity on $AdS_5 \times S^5$. It is an interesting problem to find such a description.

As the first step in this direction one could try to obtain the cubic action for supergravity. The solution of the problem would provide us with the knowledge of the relative factors in front of the two- and three-point Green functions computed in the framework of the AdS/CFT correspondence and would allow one to compare them with the ones found in the $\mathcal{N} = 4$ SYM.

The next step is to find the supergravity action up to the fourth order in the physical fields. Then one will be able to compute the four-point Green functions and to verify whether the logariphmic singularities found in [26] cancel.

Another problem to be solved is to find the spectrum and the quadratic action for the supergravity on the $AdS_5 \times \mathcal{E}_5$ background, where $\mathcal{E}_5$ is an Einstein manifold. A particularly interesting example is the manifold $\mathcal{E}_5 = T^{1,1}$. As was shown in [30] the supergravity on this background is dual to a certain large N $\mathcal{N} = 1$ superconformal field theory, which describes a non-trivial infrared fixed point of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge model.

[4] Since the derivative D_a contains the Christoffel symbols, (5.14) is not a sum of actions for spinor fields. We thank O.Rychkov for a discussion of the point.

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